Krovak Projection

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MOTTO : there are many information barriers among the cartographically oriented people. One of them is a disability to use various coordinate systems and map projections of other countries.

l. CZECH COORDINATE SYSTEM S-JTSK

S-JTSK : plane geodetic grid coordinate system called in Czech "System jednotne trigonometricke site katastralni - The System of the Unified Czech/Slovak Trigonometric Cadastral Net".

l.l History

Scale, location and orientation of the S-JTSK on the surface of the Bessel ellipsoid was derived from the results of historical Austro/Hungarian military surveys in the years 1862-98. There are 42 identical points within the Czech territory used for transformation computations. Astronomical orientation was measured only on the Hermannskogel, a trigonometric point in Austria, scale factor was derived from the length geodetic basis in Josefov.

1.2 Use, Applications

Krovak projection and national grid S-JTSK were adopted on the territory of the Czech and Slovak republics (former Czechoslovakia) in 1927. The use of this system in all geodetic surveying and cartographic activities (state mapping) is in accordance with the Act No. 200/94 Dig. of the Czech State Law.

State map series of cadastral and civil topographic maps 1:500 - 1 :200 000 are based on the JTSK System (S-JTSK). There is also a former project of the State Information System of the Czech Republic, where S-JTSK is used for territorial localisation of geographical information subjects.

2. KROVAK PROJECTION AND COORDINATE SYSTEM

While terms ***projection*** and ***coordinate system*** are often used interchangeably, they do not have the same meaning.

The **map projection** is a systematic representation of the surface of the Earth, to plane. To the identify the location of points on the Earth, a graticule or network of **longitude** **λ** (in the U.S., the abbreviations **E - East**, or **W - West** are used) and **latitude φ** (see **N - North** or **S - South**) lines have been superimposed on the surface. They are commonly referred to as meridians and parallels respectively. A projection is defined by an equation or mostly a set of equations containing several parameters where to each of those parameters has been assigned a specific value. The result is a **coordinate system**.

The **Conformal Oblique Conic Projection of Czechoslovakia** was prepared by survey engineer Josef Krovak, in the year 1922 for construction of cadastral maps and topographical maps of medium scales for the civil geodetic service of Czechoslovakia.

The projection is based on the Bessel ellipsoid of 1841, which is widely adopted in Central Europe. Longitude λ is assumed from prime Ferro meridian (an island of the Canaries), not from Greenwich meridian! The projection is conformal, so that in the projection plane meridians and parallels intersect at right angles. Angular distance between Ferro and Greenwich is given as

$$\boldsymbol{\lambda_{greenwich} = \lambda_{ferro} - 17^{o}\ 39'\ 59.7354''}$$

but round value 17º 40' was used for topographic mapping in Czechia, Slovakia and Austria.

The origin of the S-JTSK coordinate system is located in Finnish Basin, X-axis normally coincides with the meridian 42º 30' from Ferro with increasing to the South. Y-axis is perpendicular to the X-axis and increasing to the West.

2.1 The Conformal Oblique Conic Projection of former Czechoslovakia
This projection consists of four steps.

a) Conformal projection of Bessel ellipsoid to Gauss sphere
Projection ellipsoid - sphere is given by relation T_l

$$\mathbf{T_l : (\varphi, \lambda_{ferro})\ Þ\ (U,V)}$$

where input values are:
φ: latitude on the Bessel ellipsoid
$\boldsymbol{\lambda_{ferro}}$: longitude on the Bessel ellipsoid (from Ferro)

and output values
U : latitude on the Gauss spheroid
V : longitude on the Gauss spheroid
these general formulas may be written as follows :

$$\mathbf{U = f(\varphi)}$$
$$\mathbf{V = g(\lambda_{ferro})}$$

but because we use a conic projection, this relation may be inverted to

$$\mathbf{tg\,(0.5\,U + 45^\circ) = K\,\{tg^{a}\,(0.5\,\varphi + 45^\circ) * ((1-e \sin\varphi)/(1+e \sin\varphi))^{a\,e/2}\}}$$

and for value V

$$\mathbf{V} = a * \boldsymbol{\lambda_{ferro}}$$

with constants

K = 1.003419164
a = 1.000597498372
e = 0.0816968303;

b) Change from geographic to cartographic coordinates on the Gauss sphere

Purpose : change from north pole (S_p) to cartographic pole (Q) with better respecting of the borderline of the former Czechoslovak Republic. There is no change of Gauss sphere surface. This transformation is given as relation T_2

T_2 : (U,V) Þ (S,D)

where spherical triangle should be solved with the values

S : cartographic latitude on the Gauss sphere (from pole Q)
D : cartographic longitude on the Gauss sphere (from pole Q)

where so called cartographic pole Q (φ_Q , λ_Q) on the Bessel ellipsoid has coordinates

φ_Q = 59° 45' 27"
λ_Q = 42° 30' 00"

and on the Gauss sphere as the pole Q (U_Q ,V_Q), with values

U = 59° 42' 42.69689"
V = 42° 31' 31.41725"
Transformation relation T_2 is determined by formulas

$$\sin(S)=\cos(90°-U_Q)*\sin(U)+\sin(90°-U_Q)*\cos(U)*\cos(V_Q - V)$$

$$\sin(D) = \{ \cos(U) / \cos(S) \} * \sin (V_Q - V)$$

c) Conformal projection of Gauss sphere to oblique tangent cone

The aim is wrapping an oblique cone around the sphere, so that it touches the Gauss sphere surface along a standard cartographic parallel S = 78° 30' where no distance distortion is assumed. This projection is given as relation T_3

T_3 : (S,D) Þ (r, e)

where

$$r= r_o \{tg (0.5 S_o + 45°) / tg (0.5 S + 45°)\}^n$$

and

$$e= n * D$$

with constants

r_o = 1 298 039.0046 m
S_o = 78° 30'
n = sin (S_o) = 0.9799247046

where polar coordinate e is an angular distance and second polar coordinate r is a radiusvector from the cartographic pole.

d) Projection of oblique cone to grid plane S-JTSK

Polar conic coordinates (r,e) are transformed into rectangular coordinates (**X,Y**) using the relation T_4. As it was mentioned in previous text, the origin of the grid X,Y system called S-JTSK is a projection of the cartographic pole Q on plane. This point lies near Estonian Tallin in the Finnish Basin.

The X axis coinciding with the meridian of longitude $\lambda_{\mathbf{ferro}}$ = 42° 30´ increasing to the South. The Y axis is perpendicular increasing to the West. The whole Czechoslovak territory is situated in the first quadrant with only positive coordinates.

Final relation T_4 between polar and rectangular (grid S-JTSK) is given as

$$\mathbf{T_4 = (}r,e\mathbf{)}\ \text{Þ}\ \mathbf{(X,Y)}$$

where

$$\mathbf{X} = r * \mathbf{cos}\ e$$
$$\mathbf{Y} = r * \mathbf{sin}\ e$$

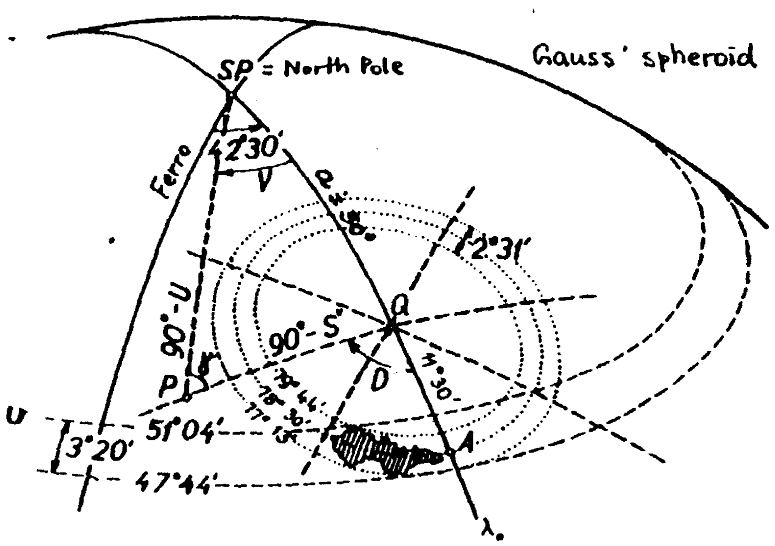

Fig 1 Krovak projection of the Czechoslovak Republic - spheroid situation

3. TEST DATA
FI = 48° 07' 46.2973"
LAMBDAF= 35° 42' 35.2147"
U = 48° 05' 29.7061 "
V = 35° 43' 52.0262"
S = 77° 43' 29.8385"
D = 21° 49' 09.0823"
RO = 1 384 345.283 m

Y = 504 691.675 m
EPSILON = 21° 22' 52.1863"
Y = 504 691.675 m
X = 1 289 068.724 m

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